

What Is Claimed Is:

1 1. A method for using a computer system to solve an unconstrained
2 interval global optimization problem specified by a function f , wherein f is a scalar
3 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots x_n)$, the method comprising:
4 receiving a representation of the function f at the computer system;
5 storing the representation in a memory within the computer system; and
6 performing an interval global optimization process to compute guaranteed
7 bounds on a globally minimum value of the function $f(\mathbf{x})$ over a subbox \mathbf{X} ;
8 wherein performing the interval global optimization process involves,
9 applying term consistency to a set of relations associated
10 with the function f over the subbox \mathbf{X} , and excluding any portion of
11 the subbox \mathbf{X} that violates any of these relations,
12 applying box consistency to the set of relations associated
13 with the function f over the subbox \mathbf{X} , and excluding any portion of
14 the subbox \mathbf{X} that violates any of these relations, and
15 performing an interval Newton step on the subbox \mathbf{X} to
16 produce a resulting subbox \mathbf{Y} , wherein the point of expansion of
17 the interval Newton step is a point \mathbf{x} within the subbox \mathbf{X} , and
18 wherein performing the interval Newton step involves evaluating
19 the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$ using interval arithmetic.

1 2. The method of claim 1, wherein applying term consistency
2 involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a term $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein
5 the term $g(x_j)$ can be analytically inverted to produce an inverse function $g^{-1}(y)$;

6 substituting the subbox \mathbf{X} into the modified equation to produce the
7 equation $g(X'_j) = h(\mathbf{X})$;
8 solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
9 intersecting X'_j with the interval X_j to produce a new subbox \mathbf{X}^+ ;
10 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
11 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
12 the size of the subbox \mathbf{X} .

1 3. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 keeping track of a smallest upper bound f_bar of the function $f(\mathbf{x})$;
4 removing from consideration any subbox \mathbf{X} for which $f(\mathbf{X}) > f_bar$; and
5 wherein applying term consistency to the f_bar relation involves applying
6 term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 4. The method of claim 3, wherein applying box consistency to the
2 set of relations involves applying box consistency to the inequality $f(\mathbf{x}) \leq f_bar$
3 over the subbox \mathbf{X} .

1 5. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 determining the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
4 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
5 removing from consideration any subbox for which any element of $\mathbf{g}(\mathbf{x})$ is
6 bounded away from zero, thereby indicating that the subbox does not include a
7 stationary point of $f(\mathbf{x})$; and

8 wherein applying term consistency to the set of relations involves applying
9 term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox
10 \mathbf{X} .

1 6. The method of claim 5, wherein applying box consistency to the
2 set of relations involves applying box consistency to each component
3 $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox \mathbf{X} .

1 7. The method of claim 1, wherein performing the interval global
2 optimization process involves:
3 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
4 function $f(\mathbf{x})$;
5 removing from consideration any subbox for which a diagonal element of
6 the Hessian is always negative, which indicates that the function f is not convex
7 and consequently does not contain a global minimum within the subbox;
8 wherein applying term consistency to the set of relations involves applying
9 term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 8. The method of claim 7, wherein applying box consistency to the
2 set of relations involves applying box consistency to each inequality
3 $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 9. The method of claim 1,
2 wherein performing the interval Newton step involves,
3 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient \mathbf{g} evaluated
4 as a function of a point \mathbf{x} over the subbox \mathbf{X} ,

5 computing an approximate inverse **B** of the center of
6 **J(x,X)**, and
7 using the approximate inverse **B** to analytically determine
8 the system **Bg(x)**, wherein **g(x)** is the gradient of the function $f(\mathbf{x})$,
9 and wherein **g(x)** includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
10 wherein applying term consistency to the set of relations involves applying
11 term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable
12 x_i ($i=1, \dots, n$) over the subbox **X**.

1 10. The method of claim 9, wherein applying box consistency to the
2 set of relations involves applying box consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$
3 ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the subbox **X**.

1 11. The method of claim 1, further comprising terminating attempts to
2 further reduce the subbox **X** when:
3 the width of **X** is less than a first threshold value; and
4 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

1 12. The method of claim 11, wherein performing the interval Newton
2 step involves:
3 computing **J(x,X)**, wherein **J(x,X)** is the Jacobian of the function **f**
4 evaluated as a function of **x** over the subbox **X**; and
5 determining if **J(x,X)** is regular as a byproduct of solving for the subbox **Y**
6 that contains values of **y** that satisfy $\mathbf{M}(\mathbf{x},\mathbf{X})(\mathbf{y}-\mathbf{x}) = \mathbf{r}(\mathbf{x})$, where
7 $\mathbf{M}(\mathbf{x},\mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x},\mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and **B** is an approximate inverse of the center of
8 **J(x,X)**.

1 13. A computer-readable storage medium storing instructions that
2 when executed by a computer cause the computer to perform a method for using a
3 computer system to solve an unconstrained interval global optimization problem
4 specified by a function f , wherein f is a scalar function of a vector
5 $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the method comprising:
6 receiving a representation of the function f at the computer system;
7 storing the representation in a memory within the computer system; and
8 performing an interval global optimization process to compute guaranteed
9 bounds on a globally minimum value of the function $f(\mathbf{x})$ over a subbox \mathbf{X} ;
10 wherein performing the interval global optimization process involves,
11 applying term consistency to a set of relations associated
12 with the function f over the subbox \mathbf{X} , and excluding any portion of
13 the subbox \mathbf{X} that violates any of these relations,
14 applying box consistency to the set of relations associated
15 with the function f over the subbox \mathbf{X} , and excluding any portion of
16 the subbox \mathbf{X} that violates any of these relations, and
17 performing an interval Newton step on the subbox \mathbf{X} to
18 produce a resulting subbox \mathbf{Y} , wherein the point of expansion of
19 the interval Newton step is a point \mathbf{x} within the subbox \mathbf{X} , and
20 wherein performing the interval Newton step involves evaluating
21 the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$ using interval arithmetic.

1 14. The computer-readable storage medium of claim 13, wherein
2 applying term consistency involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a term $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein
5 the term $g(x_j)$ can be analytically inverted to produce an inverse function $g^{-1}(y)$;

6 substituting the subbox \mathbf{X} into the modified equation to produce the
7 equation $g(X'_j) = h(\mathbf{X})$;
8 solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and
9 intersecting X'_j with the interval X_j to produce a new subbox \mathbf{X}^+ ;
10 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
11 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
12 the size of the subbox \mathbf{X} .

1 15. The computer-readable storage medium of claim 13, wherein
2 performing the interval global optimization process involves:
3 keeping track of a smallest upper bound f_bar of the function $f(\mathbf{x})$;
4 removing from consideration any subbox \mathbf{X} for which $f(\mathbf{X}) > f_bar$; and
5 wherein applying term consistency to the f_bar relation involves applying
6 term consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 16. The computer-readable storage medium of claim 15, wherein
2 applying box consistency to the set of relations involves applying box consistency
3 to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 17. The computer-readable storage medium of claim 13, wherein
2 performing the interval global optimization process involves:
3 determining the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
4 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
5 removing from consideration any subbox for which any element of $\mathbf{g}(\mathbf{x})$ is
6 bounded away from zero, thereby indicating that the subbox does not include a
7 stationary point of $f(\mathbf{x})$; and

8 wherein applying term consistency to the set of relations involves applying
9 term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox
10 \mathbf{X} .

1 18. The computer-readable storage medium of claim 17, wherein
2 applying box consistency to the set of relations involves applying box consistency
3 to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox \mathbf{X} .

1 19. The computer-readable storage medium of claim 13, wherein
2 performing the interval global optimization process involves:
3 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
4 function $f(\mathbf{x})$;

5 removing from consideration any subbox for which a diagonal element of
6 the Hessian is always negative, which indicates that the function f is not convex
7 and consequently does not contain a global minimum within the subbox;

8 wherein applying term consistency to the set of relations involves applying
9 term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 20. The computer-readable storage medium of claim 19, wherein
2 applying box consistency to the set of relations involves applying box consistency
3 to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 21. The computer-readable storage medium of claim 13,
2 wherein performing the interval Newton step involves,
3 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient \mathbf{g} evaluated
4 as a function of a point \mathbf{x} over the subbox \mathbf{X} ,

5 computing an approximate inverse **B** of the center of
6 **J(x,X)**, and
7 using the approximate inverse **B** to analytically determine
8 the system **Bg(x)**, wherein **g(x)** is the gradient of the function $f(\mathbf{x})$,
9 and wherein **g(x)** includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
10 wherein applying term consistency to the set of relations involves applying
11 term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable
12 x_i ($i=1, \dots, n$) over the subbox **X**.

1 22. The computer-readable storage medium of claim 21, wherein
2 applying box consistency to the set of relations involves applying box consistency
3 to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the
4 subbox **X**.

1 23. The computer-readable storage medium of claim 13, wherein the
2 method further comprises terminating attempts to further reduce the subbox **X**
3 when:
4 the width of **X** is less than a first threshold value; and
5 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

1 24. The computer-readable storage medium of claim 13, wherein
2 performing the interval Newton step involves:
3 computing **J(x,X)**, wherein **J(x,X)** is the Jacobian of the function **f**
4 evaluated as a function of **x** over the subbox **X**; and
5 determining if **J(x,X)** is regular as a byproduct of solving for the subbox **Y**
6 that contains values of **y** that satisfy $\mathbf{M}(\mathbf{x},\mathbf{X})(\mathbf{y}-\mathbf{x}) = \mathbf{r}(\mathbf{x})$, where

1 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
2 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.

1 25. An apparatus that solves an unconstrained interval global
2 optimization problem specified by a function f , wherein f is a scalar function of a
3 vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the apparatus comprising:
4 a receiving mechanism that is configured to receive a representation of the
5 function f ;
6 a memory for storing the representation; and
7 an interval global optimization mechanism that is configured to perform
8 an interval global optimization process to compute guaranteed bounds on a
9 globally minimum value of the function $f(\mathbf{x})$ over a subbox \mathbf{X} ;
10 a term consistency mechanism within the interval global optimization
11 mechanism that is configured to apply term consistency to a set of relations
12 associated with the function f over the subbox \mathbf{X} , and to exclude any portion of the
13 subbox \mathbf{X} that violates any of these relations;
14 a box consistency mechanism within the interval global optimization
15 mechanism that is configured to apply box consistency to the set of relations
16 associated with the function f over the subbox \mathbf{X} , and to exclude any portion of the
17 subbox \mathbf{X} that violates any of these relations; and
18 an interval Newton mechanism within the interval global optimization
19 mechanism that is configured to perform an interval Newton step on the subbox \mathbf{X}
20 to produce a resulting subbox \mathbf{Y} , wherein the point of expansion of the interval
21 Newton step is a point \mathbf{x} within the subbox \mathbf{X} , and wherein performing the interval
22 Newton step involves evaluating the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$ using
23 interval arithmetic.

1 26. The apparatus of claim 25, wherein the term consistency
2 mechanism is configured to:
3 symbolically manipulate an equation to solve for a term $g(x_j)$, thereby
4 producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein the term $g(x_j)$ can be
5 analytically inverted to produce an inverse function $g^{-1}(y)$;
6 substitute the subbox \mathbf{X} into the modified equation to produce the equation
7 $g(X'_j) = h(\mathbf{X})$;
8 solve for $X'_j = g^{-1}(h(\mathbf{X}))$; and to
9 intersect X'_j with the interval X_j to produce a new subbox \mathbf{X}^+ ;
10 wherein the new subbox \mathbf{X}^+ contains all solutions of the equation within
11 the subbox \mathbf{X} , and wherein the size of the new subbox \mathbf{X}^+ is less than or equal to
12 the size of the subbox \mathbf{X} .

1 27. The apparatus of claim 25,
2 wherein the interval global optimization mechanism is configured to,
3 keep track of a smallest upper bound f_bar of the function
4 $f(\mathbf{x})$, and to
5 remove from consideration any subbox \mathbf{X} for which
6 $f(\mathbf{X}) > f_bar$; and
7 wherein the term consistency mechanism is configured to apply term
8 consistency to the inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 28. The apparatus of claim 27, wherein the box consistency
2 mechanism is configured to apply box consistency to the inequality $f(\mathbf{x}) \leq f_bar$
3 over the subbox \mathbf{X} .

1 29. The apparatus of claim 25,

2 wherein the interval global optimization mechanism is configured to,
3 determine the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein
4 $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$), and to
5 remove from consideration any subbox for which any
6 element of $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that
7 the subbox does not include a stationary point of $f(\mathbf{x})$; and
8 wherein the term consistency mechanism is configured to apply term
9 consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox \mathbf{X} .

1 30. The apparatus of claim 29, wherein the box consistency
2 mechanism is configured to apply box consistency to each component
3 $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox \mathbf{X} .

1 31. The apparatus of claim 25,
2 wherein the interval global optimization mechanism is configured to,
3 determine diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the
4 Hessian of the function $f(\mathbf{x})$, and to
5 remove from consideration any subbox for which a
6 diagonal element of the Hessian is always negative, which
7 indicates that the function f is not convex and consequently does
8 not contain a global minimum within the subbox;
9 wherein the term consistency mechanism is configured to apply term
10 consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 32. The apparatus of claim 31, wherein the box consistency
2 mechanism is configured to apply box consistency to each inequality
3 $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 33. The apparatus of claim 25,
2 wherein the interval Newton mechanism is configured to,
3 compute the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient \mathbf{g} evaluated as
4 a function of a point \mathbf{x} over the subbox \mathbf{X} ,
5 compute an approximate inverse \mathbf{B} of the center of $\mathbf{J}(\mathbf{x}, \mathbf{X})$,
6 and to
7 use the approximate inverse \mathbf{B} to analytically determine the
8 system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and
9 wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
10 wherein the term consistency mechanism is configured to apply term
11 consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable
12 x_i ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 34. The apparatus of claim 33, wherein the box consistency
2 mechanism is configured to apply box consistency to each component
3 $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the subbox \mathbf{X} .

1 35. The apparatus of claim 25, further comprising a termination
2 mechanism that is configured to terminate attempts to further reduce the subbox \mathbf{X}
3 when:
4 the width of \mathbf{X} is less than a first threshold value; and
5 the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

1 36. The apparatus of claim 11, wherein the interval Newton
2 mechanism is configured to:

- 1 compute $\mathbf{J}(\mathbf{x}, \mathbf{X})$, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f} evaluated
2 as a function of \mathbf{x} over the subbox \mathbf{X} ; and to
3 determine if $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
4 that contains values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$, where
5 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
6 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.